Lecture 6

6.6 - Inverse Trigonometric Functions

$\arcsin x$ $(= \sin^{-1} x)$

Restricting the domain of $\sin x$ to ________ we can make it one-to-one. The range is still ________. This allows us to define an inverse sine function: $\arcsin(x)$

$D(\arcsin x) =$ $\qquad\qquad$ $R(\arcsin x) =$

Ex: Evaluate $\tan\left(\arcsin\left(\frac{\sqrt{3}}{2}\right)\right)$

Ex: Find a formula in terms of x for $\cos(\arcsin(x))$

If $\arcsin x = y \Rightarrow x = \sin y$

Differentiating gives:

So,

$$\frac{d}{dx}\left(\arcsin x\right) =$$

and the chain rule gives

$$\frac{d}{dx}\left(\arcsin(g(x))\right) =$$

which holds for x values in the domain of g and such that $-1 \leq g(x) \leq 1$.

Ex: Differentiate $f(x) = \arcsin(e^x)$. What is the domain of $f(x)$?

arccos x ($= \cos^{-1} x$)

To make $\cos x$ one-to-one, we restrict the domain to __________. This lets us define $\arccos x$.

$D(\arccos x) =$ $\qquad$ $R(\arccos x) =$

Similarly to $\arcsin x$, we can find the derivative:

$$\frac{d}{dx}\left(\arccos(x)\right) =$$

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arctan x ($= \tan^{-1} x$)

Restricting to one period of $\tan x$: ________,
we can make it one-to-one, with a range of: ________

Thus, the inverse tangent function: $\arctan x$
satisfies: $D(\arctan x) =$ $\qquad$ $R(\arctan x) =$

Ex: Evaluate $\sin\left(\arctan\left(\frac{1}{\sqrt{3}}\right)\right)$.

Writing $y = \arctan x$, we have $\tan y = x$.

Differentiating:

<u>Ex</u>: Let $f(x) = \arctan(\ln(2-x^2))$. What is the domain of $f(x)$? What is $f'(x)$?

Integration

We can reverse the derivatives we found above to get:

$$\int \frac{1}{\sqrt{1-x^2}}\,dx =$$

$$\int \frac{1}{1+x^2}\,dx =$$

Ex: Compute

$$\int \frac{1}{x^2+a^2}\,dx$$

Ex: Compute

$$\int \frac{e^{2x}}{\sqrt{1-e^{4x}}}\,dx$$